



POSTAL BOOK PACKAGE 2025

ELECTRONICS ENGINEERING

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CONVENTIONAL Practice Sets

CONTENTS

ELECTROMAGNETICS

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Vector Analysis

Q1 For a vector field \vec{A} , show explicitly that $\nabla \cdot \nabla \times \vec{A} = 0$; that is, the divergence of the curl of any vector field is zero.

Solution:

$$\begin{aligned}\nabla \cdot \nabla \times \vec{A} &= \frac{\partial}{\partial x} \cdot \hat{a}_x + \frac{\partial}{\partial y} \cdot \hat{a}_y + \frac{\partial}{\partial z} \cdot \hat{a}_z \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \\ &= \frac{\partial}{\partial x} \cdot \hat{a}_x + \frac{\partial}{\partial y} \cdot \hat{a}_y + \frac{\partial}{\partial z} \cdot \hat{a}_z \left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{a}_x - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) \hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{a}_z \right] \\ &= \frac{\partial}{\partial x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) - \frac{\partial}{\partial y} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\ &= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0\end{aligned}$$

Because $\frac{\partial^2 A_z}{\partial x \partial y} = \frac{\partial^2 A_z}{\partial y \partial x}$ and so on.

Q2 If $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, then show that the vector $\vec{A} = xz \hat{a}_x + z^2 \hat{a}_y + yz \hat{a}_z$ satisfy this vector identity.

Solution:

We have to verify, $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$... (i)

Taking L.H.S.

$$\begin{aligned}\nabla \times \vec{A} &= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & z^2 & yz \end{vmatrix} \\ &= \left(\frac{\partial}{\partial x} z^2 - \frac{\partial}{\partial y} xz \right) \hat{a}_z + \left(\frac{\partial}{\partial y} (yz) - \frac{\partial}{\partial z} (z^2) \right) \hat{a}_x - \left(\frac{\partial}{\partial x} yz - \frac{\partial}{\partial z} xz \right) \hat{a}_y \\ &= (z - 2z) \hat{a}_x + x \hat{a}_y = -z \hat{a}_x + x \hat{a}_y \quad \dots \text{(ii)}\end{aligned}$$

\therefore

$$\begin{aligned}\nabla \times (\nabla \times \vec{A}) &= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z & x & 0 \end{vmatrix} \\ &= 0 \cdot \hat{a}_x - (+1) \hat{a}_y + \hat{a}_z = -\hat{a}_y + \hat{a}_z \quad \dots \text{(iii)}\end{aligned}$$

now considering R.H.S

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(z^2) + \frac{\partial}{\partial z} yz = z + y$$

$$\begin{aligned} \nabla(\nabla \cdot \vec{A}) &= \frac{\partial}{\partial x}(\nabla \cdot \vec{A})\hat{a}_x + \frac{\partial}{\partial y}(\nabla \cdot \vec{A})\hat{a}_y + \frac{\partial}{\partial z}(\nabla \cdot \vec{A})\hat{a}_z = \frac{\partial}{\partial x}(z+y)\hat{a}_x + \frac{\partial}{\partial y}(z+y)\hat{a}_y + \frac{\partial}{\partial z}(z+y)\hat{a}_z \\ &= \hat{a}_y + \hat{a}_z \end{aligned} \quad \dots(\text{iv})$$

$$\nabla^2 \vec{A} = \nabla^2 A_x \hat{a}_x + \nabla^2 A_y \hat{a}_y + \nabla^2 A_z \hat{a}_z$$

$$\text{and } \nabla^2 \vec{A} = \left[\frac{\partial^2}{\partial x^2}(xz) + \frac{\partial^2}{\partial y^2}(xz) + \frac{\partial^2}{\partial z^2}(xz) \right] \hat{a}_x + \left[\frac{\partial^2}{\partial x^2}(z^2) + \frac{\partial^2}{\partial y^2}(z^2) + \frac{\partial^2}{\partial z^2}(z^2) \right] \hat{a}_y + \left[\frac{\partial^2}{\partial x^2}(yz) + \frac{\partial^2}{\partial y^2}(yz) + \frac{\partial^2}{\partial z^2}(yz) \right] \hat{a}_z$$

$$\nabla^2 \vec{A} = 2\hat{a}_y \quad \dots(\text{v})$$

From equation (iv) and (v), we get,

$$\nabla \cdot (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \hat{a}_y + \hat{a}_z - 2\hat{a}_y = \hat{a}_z - \hat{a}_y \quad \dots(\text{vi})$$

∴ From equation (iii) and (vi),

$$\text{RHS} = \text{LHS} = \hat{a}_z - \hat{a}_y$$

Hence Proved.

Q3 If $\vec{F} = x^2 y \hat{a}_x - 2z \hat{a}_y + (3z^2 + xy) \hat{a}_z$, find $\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})]$.

Solution:

$$\text{Given } \vec{F} = x^2 y \hat{a}_x - 2z \hat{a}_y + (3z^2 + xy) \hat{a}_z$$

Let us calculate $\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})]$ step by step.

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x^2 y) - \frac{\partial}{\partial y}(2z) + \frac{\partial}{\partial z}(3z^2 + xy) = 2xy + 6z$$

$$\nabla \cdot (\nabla \cdot \vec{F}) = \nabla \cdot (2xy + 6z)$$

$$= \frac{\partial}{\partial x}(2xy + 6z)\hat{a}_x + \frac{\partial}{\partial y}(2xy + 6z)\hat{a}_y + \frac{\partial}{\partial z}(2xy + 6z)\hat{a}_z = 2y\hat{a}_x + 2x\hat{a}_y + 6\hat{a}_z$$

$$\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})] = \nabla \times (2y\hat{a}_x + 2x\hat{a}_y + 6\hat{a}_z)$$

$$= \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 2x & 6 \end{bmatrix} = \left(\frac{\partial(6)}{\partial y} - \frac{\partial(2x)}{\partial z} \right) \hat{a}_x - \left(\frac{\partial(6)}{\partial x} - \frac{\partial(2y)}{\partial z} \right) \hat{a}_y + \left(\frac{\partial(2x)}{\partial x} - \frac{\partial(2y)}{\partial y} \right) \hat{a}_z$$

$$= 0\hat{a}_x - 0\hat{a}_y + (2 - 2)\hat{a}_z = 0$$

Note that $\nabla \times \nabla \cdot \vec{F} = 0$

Q4 E and F are vector fields given by $E = 2x \hat{a}_x + \hat{a}_y + yz \hat{a}_z$ and $F = xy \hat{a}_x - y^2 \hat{a}_y + xy z \hat{a}_z$. Determine

(a) $|E|$ at $(1, 2, 3)$

(b) The component of E along F at $(1, 2, 3)$

(c) A vector perpendicular to both E and F at $(0, 1, -3)$ whose magnitude is unity.

Solution:

$$(a) \quad \vec{E} = 2x \hat{a}_x + \hat{a}_y + yz \hat{a}_z ; \quad \text{At point } (1, 2, 3) \Rightarrow \vec{E} = 2\hat{a}_x + \hat{a}_y + 6\hat{a}_z$$

$$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} = 6.403$$

$$(b) \quad \vec{F} = xy \hat{a}_x - y^2 \hat{a}_y + xyz \hat{a}_z ; \quad \text{At } (1, 2, 3), \vec{F} = 2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z$$

\therefore The component of \vec{E} along \vec{F}

$$E_F = (\vec{E} \cdot \hat{a}_F) \hat{a}_F \quad \left[\hat{a}_F = \frac{\vec{F}}{|\vec{F}|} \right]$$

$$= \frac{(\vec{E} \cdot \vec{F})}{|\vec{F}|^2} \vec{F} = \frac{36}{56} (2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) = 1.286\hat{a}_x - 2.571\hat{a}_y + 3.857\hat{a}_z$$

$$(c) \quad \text{At } (0, 1, -3) \quad \vec{E} = 0\hat{a}_x + \hat{a}_y - 3\hat{a}_z$$

$$\vec{F} = 0\hat{a}_x - \hat{a}_y + 0\hat{a}_z$$

$$E \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = -3\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$a_{E \times F} = \pm \frac{\vec{E} \times \vec{F}}{|\vec{E} \times \vec{F}|} = \pm \hat{a}_x$$

Q5 Let $\vec{H} = 5\rho \sin\phi \hat{a}_\rho - \rho z \cos\phi \hat{a}_\phi + 2\rho \hat{a}_z$ A/m. At point $P(2, 30^\circ, -1)$ find:

(a) a unit vector along \vec{H} .

(b) the component of \vec{H} parallel to \hat{a}_x .

(c) the component of \vec{H} normal to $\rho = 2$.

(d) the component of \vec{H} tangential to $\phi = 30^\circ$.

Solution:

At P , $\rho = 2, \phi = 30^\circ, z = -1$

$$\vec{H} = 10 \sin 30^\circ \hat{a}_\rho + 2 \cos 30^\circ \hat{a}_\phi + 4 \hat{a}_z = 5\hat{a}_\rho + 1.732 \hat{a}_\phi + 4\hat{a}_z \text{ A/m}$$

(a) Unit vector along \vec{H} ,

$$\hat{a}_H = \frac{5\hat{a}_\rho + 1.732 \hat{a}_\phi + 4\hat{a}_z}{\sqrt{5^2 + 1.732^2 + 4^2}} = 0.7538 \hat{a}_\rho + 0.2611 \hat{a}_\phi + 0.603 \hat{a}_z$$

(b) The relation between Cartesian coordinates (A_x, A_y, A_z) and cylindrical coordinates (A_ρ, A_ϕ, A_z) is given as:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$H_x = H_\rho \cos\phi - H_\phi \sin\phi \\ = 5\rho \sin\phi \cos\phi - \rho z \cos\phi \sin\phi$$

At P , $\rho = 5, \phi = 30^\circ, z = -1$

$$H_x = H_x \hat{a}_x = (25 \sin 30^\circ \cos 30^\circ + 5 \sin 30^\circ \cos 30^\circ) \hat{a}_x = 13 \hat{a}_x \text{ A/m}$$

(c) Normal to $\rho = 2$ is $\vec{H}_n = \vec{H}_\rho \hat{a}_\rho$

i.e. $\vec{H}_n = 0.7538 \hat{a}_\rho \text{ A/m}$

(d) Tangential to $\phi = 30^\circ$

$$H_t = H_\rho \cdot \hat{a}_\rho + H_z \hat{a}_z = 0.7538 \hat{a}_\rho + 0.603 \hat{a}_z \text{ A/m}$$

Q6 Find the rate at which the scalar function, $V = r^2 \sin 2\phi$, in cylindrical co-ordinates, increases in the direction of the vector $A = \hat{a}_r + \hat{a}_\phi$ at the point having co-ordinates $(2, \pi/4, 0)$.

Solution:

As we know that, Gradient is a vector that represents both the magnitude and the direction of maximum space rate of the increase of the scalar function i.e.

$$\text{grad } V = \nabla V = \frac{dV}{dn} \hat{a}_n \quad \dots(i)$$

But in cylindrical coordinate system, the grad V can be defined as,

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z \quad \dots(ii)$$

For the case under consideration, the quantity required is,

$$\nabla V \cdot \hat{a}_A = \nabla V \cdot \frac{\vec{A}}{|\vec{A}|} \quad \text{at } (2, \pi/4, 0)$$

From equation (ii) we have,

$$\nabla V = \frac{\partial}{\partial r}(r^2 \sin 2\phi) \hat{a}_r + \frac{1}{r} \frac{\partial}{\partial \phi}(r^2 \sin 2\phi) \hat{a}_\phi + \frac{\partial}{\partial z}(r^2 \sin 2\phi) \hat{a}_z.$$

or,

$$\nabla V = 2r \sin 2\phi \hat{a}_r + 2r \cos 2\phi \hat{a}_\phi$$

Now,

$$\nabla V \cdot \vec{A} = (2r \sin 2\phi \hat{a}_r + 2r \cos 2\phi \hat{a}_\phi) \cdot (\hat{a}_r + \hat{a}_\phi)$$

or,

$$\nabla V \cdot \vec{A} = 2r \sin 2\phi + 2r \cos 2\phi \quad \dots(iii)$$

Also,

$$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

∴

$$\begin{aligned} \nabla V \cdot \hat{a}_A &= \frac{\nabla V \cdot \vec{A}}{|\vec{A}|} = \frac{1}{\sqrt{2}} [2r \sin 2\phi + 2r \cos 2\phi] \\ &= (\sqrt{2} r \sin 2\phi + \sqrt{2} r \cos 2\phi) \end{aligned}$$

Now,

$$(\nabla V \cdot \hat{a}_A)_{\text{at}(2, \pi/4, 0)} = \sqrt{2} \times 2 \times \sin\left(2 \times \frac{\pi}{4}\right) + \sqrt{2} \times 2 \times \cos\left(2 \times \frac{\pi}{4}\right) = 2\sqrt{2}$$

Q7 Write down the Divergence theorem. An electric field at point P , expressed in cylindrical co-ordinate system is given by,

$$\vec{E} = 3r^2 \sin \phi \hat{a}_r + 2r^2 \cos \phi \hat{a}_\phi$$

Find the value of divergence of the field if the location of point ' P ' is given by $(9, 9, 9)$ in Cartesian co-ordinate system.

Solution:

Divergence Theorem: The divergence theorem states that, the volume integral of the divergence of a vector field ' A ', taken over any volume V is equal to the surface integral of A , taken over the closed surface that is

$$\int_V (\nabla \cdot \vec{A}) dV = \oint_S \vec{A} \cdot ds$$

Given that,

$$\vec{E} = 3r^2 \sin \phi \hat{a}_r + 2r^2 \cos \phi \hat{a}_\phi$$

In cylindrical coordinate system,

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r}(rE_r) + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}(3r^3 \sin \phi) + \frac{1}{r} \frac{\partial}{\partial \phi}(2r^2 \cos \phi)$$

$$= \frac{1}{r}(9r^2 \sin\phi) + \frac{1}{r}(2r^2(-\sin\phi)) = 9r\sin\phi - 2r\sin\phi$$

$$\nabla \cdot \vec{E} = 7r\sin\phi$$

Converting into Cartesian co-ordinates

$$r = \sqrt{x^2 + y^2}$$

and

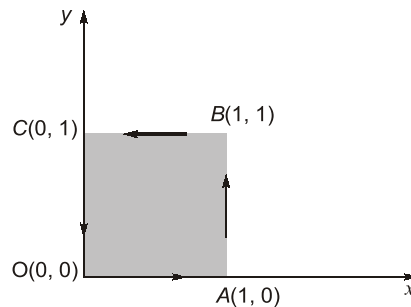
$$\sin\phi = \frac{y}{\sqrt{x^2 + y^2}}$$

∴

$$\nabla \cdot \vec{E} = 7\sqrt{x^2 + y^2} \left(\frac{y}{\sqrt{x^2 + y^2}} \right) = 7\sqrt{9^2 + 9^2} \cdot \frac{9}{\sqrt{9^2 + 9^2}} = 7 \times 9 = 63$$

$$\nabla \cdot \vec{E} = 63$$

- Q8** (a) Given vector $\vec{F} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$, find circulation of \vec{F} along a closed path $OABC$ as shown in figure below:



- (b) Check the above result using Stoke's theorem. According to Stoke's theorem

$$\oint \vec{F} \cdot d\vec{l} = \int (\nabla \times \vec{F}) \cdot d\vec{s}$$

Solution:

- (a) In a closed path the circulation of vector \vec{F} is given as

$$\oint \vec{F} \cdot d\vec{l} = \int_{OA} + \int_{AB} + \int_{BC} + \int_{CO} \vec{F} \cdot d\vec{l}$$

Given $\vec{F} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$; $d\vec{l} = dx\hat{a}_x + dy\hat{a}_y$; and $\vec{F} \cdot d\vec{l} = F_x dx + F_y dy = x^2y dx + 2xy^2 dy$

For path OA , $dy = 0$, $y = 0$, $F = 0$

and,

$$\int_{OA} \vec{F} \cdot d\vec{l} = \int_{OA} x^2y dx = 0$$

For path AB , $dx = 0$, $x = 1$, $F = y\hat{a}_x + 2y^2\hat{a}_y$

and

$$\int_{AB} \vec{F} \cdot d\vec{l} = \int_0^1 2y^2 dy = \frac{2y^3}{3} = \frac{2}{3}(1-0) = \frac{2}{3}$$

For path BC ,

$$dy = 0, \quad y = 1, \quad \vec{F} = x^2\hat{a}_x + 2x\hat{a}_y$$

and,

$$\int_{BC} \vec{F} \cdot d\vec{l} = \int_1^0 x^2 dx = \frac{x^3}{3} = \frac{-1}{3}$$